7.1.1 → ← 7.1.2

Chap 7 Techniques of Integration

Sec 7.1 Integration by Parts

Introduction & Objectives

We know that the integral of a product is not product of integral;

$$\int f(x)g(x)\,dx \neq \left[\int f(x)\,dx\right]\left[\int g(x)\,dx\right]$$

However, we will learn a technique for evaluating integrals of the form $\int f(x)g'(x)\,dx$. This techniques allows us to transfer the derivative from one func in the product to another. This will be very helpful if the new integral is much simpler.

We will use this technique to evaluate integrals like

$\int xe^{x}\,dx$, $\int x^2\sin x$, $\int e^{x}\sin x\,dx$, $\int \ln x\,dx$, and

$\int \tan^{-1}x\,dx$.

Integration by Parts

Suppose $u = u(x)$ & $v = v(x)$ are differentiable functions

$$\frac{d}{dx}(uv) = \frac{du}{dx}v + u\frac{dv}{dx}$$

$$d(uv) = v\,du + u\,dv$$

$$u\,dv = d(uv) - v\,du$$

$$\int u\,dv = \int d(uv) - \int v\,du$$

$$\underline{\int u\,dv = uv - \int v\,du}$$

7.1.3 → ← 7.1.4

letting $u = f(x)$ & $v = g(x)$. We have $du = f'(x)dx$ and $dv = g'(x)\,dx$ and the last formula becomes

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

Integration by Parts

$$\int u\,dv = uv - \int v\,du$$

If $u(a) = u_1$, $v(a) = v_1$, $u(b) = u_2$, $v(b) = v_2$, then

$$\int_{v_1}^{v_2} u\,dv = (uv)\Big|_{u=u_1, v=v_1}^{u=u_2, v=v_2} - \int_{u_1}^{u_2} v\,du$$

$$\int f(x)g'(x)\,dx = f(x)g(x) - \int f'(x)g(x)\,dx$$

$$\int_a^b f(x)g'(x)\,dx = [f(x)g(x)]\Big|_a^b - \int_a^b f'(x)g(x)\,dx$$

Ex Evaluate the following.

1. $\int x e^{2x}\,dx$

2. $\int_1^e x\ln x\,dx$

7.1.5 → ← 7.1.6

Ex Evaluate the following.

1. $\int x^2 e^x \, dx$

2. $\int_0^{\pi/2} x^2 \sin x \, dx$

Ex Evaluate the following.

1. $\int \ln x \, dx$

2. $\int_0^1 \tan^{-1} x \, dx$

7.1.7 → ← 7.1.8

Ex Evaluate $\int e^x \sin x \, dx$.

Let $u = e^x$ & $dv = \sin x \, dx$. Then
$du = e^x dx$ & $v = -\cos x$

$\int e^x \sin x \, dx = -e^x \cos x - \int -\cos x \, e^x dx$
$= -e^x \cos x + \int e^x \cos x \, dx \Rightarrow$

Let $u = e^x$ & $dv = \cos x \, dx$. Then
$du = e^x dx$ & $v = \sin x \, dx$

$=$

7.1.9 → | ← 7.1.10

Ex Evaluate $\int e^x \sin x \, dx$

Let $u = \sin x$ & $dv = e^x dx$. Then
$du = \cos x \, dx$ & $v = e^x$

$\int e^x \sin x \, dx = e^x \sin x - \int e^x \cos x \, dx =$

Let $u = \cos x$ & $dv = e^x dx$. Then
$du = -\sin x \, dx$ & $V = e^x$

$= e^x \sin x - \left[e^x \cos x - \int e^x (-\sin x) dx \right] + C$

$= e^x \sin x - e^x \cos x - \int e^x \sin x \, dx + C$

$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx + C$

$2 \int e^x \sin x \, dx = e^x \sin x - e^x \cos x + C$

$\int e^x \sin x \, dx = \frac{1}{2} e^x \sin x - \frac{1}{2} e^x \cos x + C$